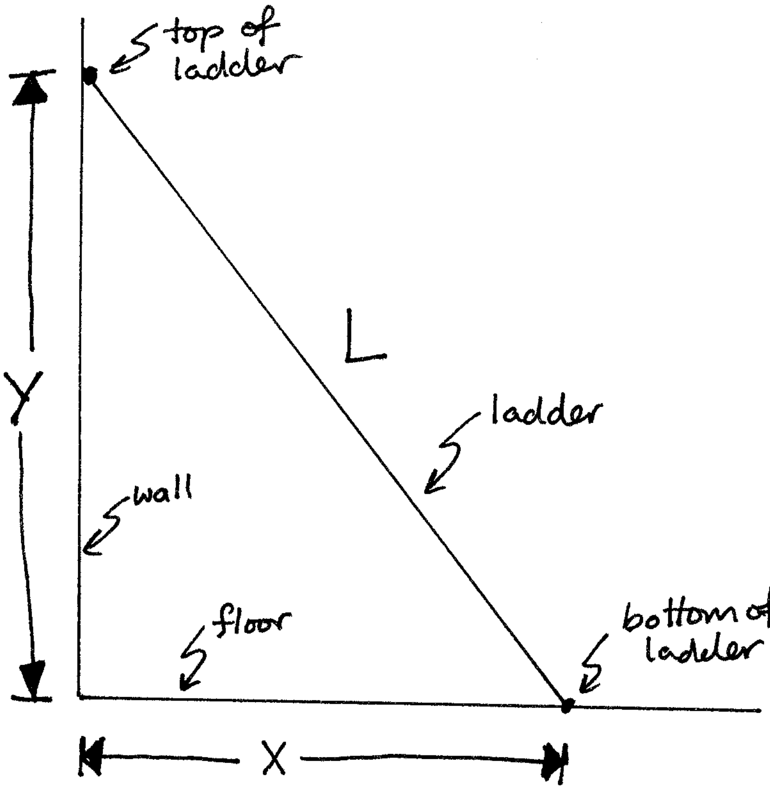
#1.



Define L to be the length of the ladder.
Define $x = x(t)$ to be the distance between the bottom of the ladder and the wall, at time t.
Define $y = y(t)$ to be the distance between the top of the ladder and the floor, at time t.
We are given that

$$\left.\frac{dx}{dt}\right|_{x=8} = 3 \quad \text{and} \quad \left.\frac{dy}{dt}\right|_{x=8} = -4.$$

(* Note that since y is decreasing in length as the ladder moves down the wall we must have $\frac{dy}{dt}$ being negative. Similarly as x is increasing must have $\frac{dx}{dt}$ being positive)

#1 continued. By Phythagoras have,

$$x^2+y^2=L^2.$$

When $x=8$, have $8^2+y^2=L^2$, or $y^2=L^2-64$.
Since $y>0$ we have $y=\sqrt{L^2-64}$.

Now implicitly differentiate $x^2+y^2=L^2$ with respect to time t. We get

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

Substituting $x=8$ & $y=\sqrt{L^2-64}$ into this we find

$$2(8)(3)+2\sqrt{L^2-64}\,(-4)=0$$

Solving for L we have,

$$\sqrt{L^2-64}=\frac{-2(8)(3)}{2(-4)}=(2)(3)=6$$

So $L^2-64=36$

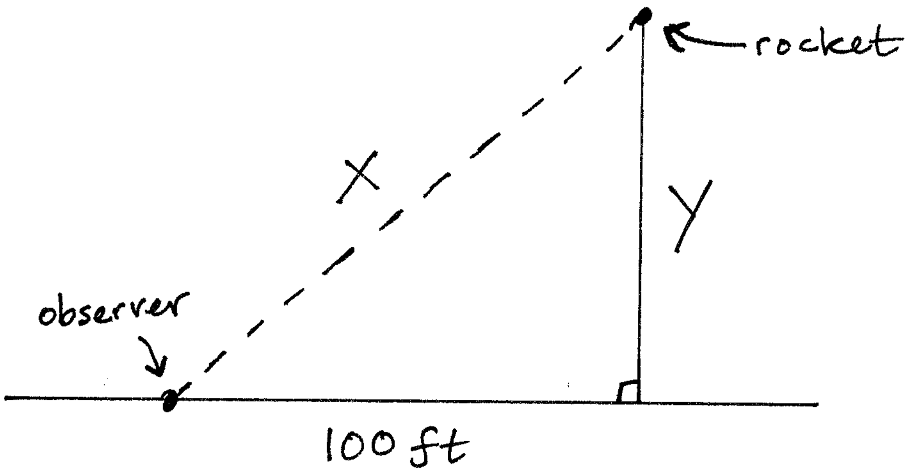
$L^2=100$

$\underline{\underline{L=10\text{ ft}}}$

$\boxed{\text{The length of the ladder is } 10\text{ ft}}$

(answer)

#2.



Let $X = X(t)$ be the distance between the rocket and the observer at time t.

Let $y = y(t)$ be the distance between the rocket and the ground at time t.

We are given $\frac{dy}{dt} = 60$ and we want to find $\left.\frac{dx}{dt}\right|_{y=300}$. Well by Pythagoras we know

$$100^2 + y^2 = x^2.$$

Note when $y = 300$ we have $x^2 = 300^2 + 100^2$ so $x^2 = 100\,000 = 10^5$, $x = \sqrt{10^5} = 100\sqrt{10}$.

Implicitly differentiating $100^2 + y^2 = x^2$ with respect to time t gives

$$0 + 2y\frac{dy}{dt} = 2x\frac{dx}{dt}.$$

Substituting $y = 300$, $x = 100\sqrt{10}$ in this gives

#2. continued.

$$2(300)\left(\left.\frac{dy}{dt}\right|_{y=300}\right) = 2(100\sqrt{10})\left(\left.\frac{dx}{dt}\right|_{y=300}\right).$$

Since $\frac{dy}{dt} = 60$ always, in particular $\left.\frac{dy}{dt}\right|_{y=300} = 60.$

Substituting this in gives

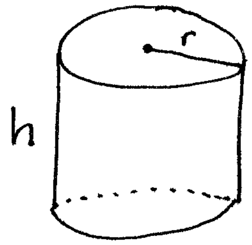
$$2(300)(60) = 2(100\sqrt{10})\left(\left.\frac{dx}{dt}\right|_{y=300}\right)$$

So $\left.\frac{dx}{dt}\right|_{y=300} = \frac{2(300)(60)}{2(100\sqrt{10})} = \frac{180}{\sqrt{10}} \approx 56.92 \text{ ft/s}.$

$$\boxed{\text{When the rocket is 300 ft off the ground the rate of change between the observer and the rocket is 56.92 ft/s.}}$$

(answer)

#3.



- Given that there is a positive constant c such that

$$\frac{dr}{dt} = c.$$

- Given that there are constants a and b such that $h = ar + b$. (this is what it means to be a linear function)
- Given $\frac{dh}{dt} = 3\frac{dr}{dt} = 3c$.

Since $h = ar + b$ have $\frac{dh}{dt} = a\frac{dr}{dt} + 0 = ac$.

So $ac = 3c$. Since $c \neq 0$ get $a = 3$.
Therefore $h = 3r + b$.

- Given that when $r = 1$, $h = 6$. So substituting this in $h = 3r + b$ gives $6 = 3(1) + b$, and so $b = 3$.
Therefore $h = 3r + 3$.

- Let V be the volume of the cylinder. So $V = \pi r^2 h$.
Since $h = 3r + 3$ we see $V = \pi r^2(3r+3) = 3\pi(r^3 + r^2)$.
Differentiating with respect to t gives

$$\frac{dV}{dt} = 3\pi\left(3r^2\frac{dr}{dt} + 2r\frac{dr}{dt}\right)$$

$$\frac{dV}{dt} = 3\pi(3r^2c + 2rc)$$

$$\frac{dV}{dt} = 3\pi rc(3r + 2).$$

- Given $\left.\frac{dV}{dt}\right|_{r=6} = 1.$ So

$$1 = \left.\frac{dV}{dt}\right|_{r=6} = 3\pi(6)c(3\cdot 6+2)$$

$1 = 360\pi c$, and so $c = \frac{1}{360\pi}$.

- The question asks us to find $\left.\frac{dV}{dt}\right|_{r=36}$. Well,

$$\left.\frac{dV}{dt}\right|_{r=36} = 3\pi(36)\left(\frac{1}{360\pi}\right)(3(36)+2)$$

$$= 33.$$

$$\boxed{\text{When the radius is 36 ft the volume of the cylinder is changing at a rate of } 33\ \text{ft}^3/\text{s}.}$$

(answer)