#1. Use substitution. Define $u = \sin x$

then $du = \cos x \, dx$

so $dx = \dfrac{du}{\cos x}$

$$\int e^{\sin x} \cos x \, dx = \int e^{u} (\cancel{\cos x}) \left(\frac{du}{\cancel{\cos x}} \right)$$

$$= \int e^{u} \, du \qquad \text{(because the } \cos x \text{ cancel)}$$

$$= e^{u} + C$$

$$= e^{\sin x} + C$$

So $\boxed{\int e^{\sin x} \cos x \, dx = e^{\sin x} + C}$ (answer)

#2. Use Integration by Parts. The formula is

$$\int u \, dv = uv - \int v \, du$$

Take $u = e^{x}$ $\quad v = \sin x$

$du = e^{x} \, dx$ $\quad dv = \cos x \, dx$

Therefore

$$\int e^{x} \cos x \, dx = \int u \, dv = uv - \int v \, du$$

$$= e^{x} \sin x - \int (\sin x)(e^{x} \, dx)$$

#2. continued. We have shown

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$$

Now use Integration by parts again on the right hand integral.

Take $u = e^x$ $\qquad v = -\cos x$

$du = e^x \, dx \qquad dv = \sin x \, dx$

Therefore,

$$\int e^x \cos x \, dx = e^x \sin x - \left[e^x(-\cos x) - \int (-\cos x)(e^x \, dx) \right]$$

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

Moving the right hand integral to the left hand side of the equal sign gives:

$$\int e^x \cos x \, dx + \int e^x \cos x \, dx = e^x \sin x + e^x \cos x$$

So $\quad 2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x$

or $\quad \int e^x \cos x \, dx = \frac{1}{2} (e^x \sin x + e^x \cos x)$

Notice that this equation has the form

$a = b + c - a.$

We want to solve for a. So

$a + a = b + c$

$2a = b + c$

$a = \frac{b+c}{2}$

or $a = \frac{1}{2}(b+c).$

Therefore

$$\boxed{\int e^x \cos x \, dx = \frac{e^x}{2} (\sin x + \cos x) + C}$$

(answer)

#3. Use Substitution.

Define $u = 2x^3+1$

then $du = 6x^2\,dx$

so $dx = \dfrac{du}{6x^2}$

$$\int \frac{x^2}{\sqrt{2x^3+1}}\,dx = \int \frac{x^2}{\sqrt{u}}\left(\frac{du}{6x^2}\right)$$

$$= \int \frac{1}{\sqrt{u}}\,\frac{du}{6}$$ (because the x^2's cancel)

$$= \frac{1}{6}\int u^{-1/2}\,du$$

$$= \frac{1}{6}(2u^{1/2}) + C$$

$$= \frac{1}{3}(2x^3+1)^{1/2} + C.$$

Therefore $$\boxed{\int \frac{x^2}{\sqrt{2x^3+1}}\,dx = \frac{1}{3}\sqrt{2x^3+1} + C}$$ (answer)

#4. Use Substitution.

Define $u = x + \cos x$

then $du = (1-\sin x)\,dx$

so $dx = \dfrac{du}{1-\sin x}$

$$\int \frac{1-\sin x}{x+\cos x}\,dx = \int \frac{1-\sin x}{u}\left(\frac{du}{1-\sin x}\right)$$

$$= \int \frac{1}{u}\,du = \int u^{-1}\,du = \ln|u| + C$$

Therefore $$\boxed{\int \frac{1-\sin x}{x+\cos x}\,dx = \ln|x+\cos x| + C}$$ (answer).

#5. Use Integration by Parts.

Take $u = \ln x$ $\quad v = x$

$du = \frac{1}{x}\,dx$ $\quad dv = 1\,dx$

So $\int \ln x\,dx = \int u\,dv = uv - \int v\,du$

$$= x\ln x - \int x\left(\tfrac{1}{x}\,dx\right)$$

$$= x\ln x - \int 1\,dx$$

$$= x\ln x - x + C.$$

$\boxed{\int \ln x\,dx = x\ln x - x + C}$ (answer)

#6. Use Integration by parts.

Take $u = x^2$ $\quad v = e^x$

$du = 2x\,dx$ $\quad dv = e^x\,dx$

$\int x^2 e^x\,dx = \int u\,dv = uv - \int v\,du$

$$= x^2e^x - \int e^x(2x\,dx)$$

$$= x^2e^x - 2\int xe^x\,dx.$$

Now use Integration by parts again on the right hand integral: $\int xe^x\,dx$.

#6 continued. Take $u = x$ $\quad v = e^x$

$du = 1\,dx$ $\quad dv = e^x\,dx$

So $\int x^2 e^x\,dx = x^2 e^x - 2\int x e^x\,dx$

$$= x^2e^x - 2\left(xe^x - \int e^x\,dx\right)$$

$$= x^2e^x - 2xe^x + 2\int e^x\,dx$$

$$= x^2e^x - 2xe^x + 2e^x + C.$$

$$\boxed{\int x^2e^x\,dx = x^2e^x - 2xe^x + 2e^x + C}$$ (answer)

#7. Use substitution. (can also use integration by parts).

Define $u = x+1$, so $x = u-1$, and $dx = 1\,du$.

$$\int x\sqrt{x+1}\,dx = \int (u-1)\sqrt{u}\,du = \int u^{3/2} - u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{15}\left(3(x+1)^{5/2} - 5(x+1)^{3/2}\right) + C$$

$$= \frac{2(x+1)^{3/2}}{15}\left(3(x+1) - 5\right) + C$$

$$= \frac{2(x+1)^{3/2}}{15}(3x-2) + C$$

$$\boxed{\int x\sqrt{x+1}\,dx = \frac{2(x+1)^{3/2}(3x-2)}{15} + C}$$

#8. Recall the formula $\cos 2\theta = 2\cos^2\theta - 1$. If we solve for $\cos^2\theta$ it becomes

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta).$$

Therefore by setting $\theta = x$ we see

$$\cos^4 x = (\cos^2 x)^2 = \left(\frac{1}{2}(1+\cos 2x)\right)^2$$

$$= \frac{1}{4} + \cos 2x + \frac{1}{4}\cos^2 2x$$

The above formula implies, by setting $\theta = 2x$, that

$$\cos^2 2x = \frac{1}{2}(1+\cos(2(2x))) = \frac{1}{2}(1+\cos 4x).$$

So

$$\cos^4 x = \frac{1}{4} + \cos 2x + \frac{1}{4}\left(\frac{1}{2}(1+\cos 4x)\right)$$

$$= \frac{1}{4} + \cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x.$$

Thus,

$$\int \cos^4 x \, dx = \int \frac{3}{8} + \cos 2x + \frac{1}{8}\cos 4x \, dx$$

$$= \frac{3}{8}x + \frac{1}{2}\sin 2x + \frac{1}{32}\sin 4x + C.$$

$$\boxed{\int \cos^4 x \, dx = \frac{3}{8}x + \frac{1}{2}\sin 2x + \frac{1}{32}\sin 4x + C}$$ (answer)

#9. Use partial fractions. First factor the bottom $x^2 - 3x - 10 = (x-5)(x+2)$. So we need to find real numbers A and B such that

$$\frac{2x}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}$$

Multiplying both sides by $(x-5)(x+2)$ we get

$$2x = A(x+2) + B(x-5).$$

Substituting $x=-2$ and $x=5$ into this we get the two equations

$$-4 = 0 + B(-7) \Rightarrow B = 4/7$$

and $$10 = A(7) + 0 \Rightarrow A = 10/7$$

So

$$\frac{2x}{x^2-3x-10} = \frac{10/7}{x-5} + \frac{4/7}{x+2}$$

Therefore

$$\int \frac{2x}{x^2-3x-10}\,dx = \frac{10}{7}\int \frac{1}{x-5}\,dx + \frac{4}{7}\int \frac{1}{x+2}\,dx$$

$$= \frac{10}{7}\ln|x-5| + \frac{4}{7}\ln|x+2| + C.$$

$$\boxed{\int \frac{2x}{x^2-3x-10}\,dx = \frac{10}{7}\ln|x-5| + \frac{4}{7}\ln|x+2| + C}$$

(answer)

#10. Must factor the bottom and use partial fractions.

Notice that $x=-1$ is a root of $x^3+8x^2+17x+10$

since $(-1)^3+8(-1)^2+17(-1)+10 = -1+8-17+10 = 0$.

So $x+1$ is a factor of $x^3+8x^2+17x+10$.

$$\begin{array}{r|l}
 & x^2+7x+10 \\
\hline
x+1 & x^3+8x^2+17x+10 \\
 & \underline{x^3+x^2} \\
 & 7x^2+17x \\
 & \underline{7x^2+7x} \\
 & 10x+10 \\
 & \underline{10x+10} \\
 & 0
\end{array}$$

So

$x^3+8x^2+17x+10$

$= (x+1)(x^2+7x+10)$

$= (x+1)(x+2)(x+5)$

We must find real numbers A, B, C such that

$$\frac{1}{(x+1)(x+2)(x+5)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+5}$$

Multiply both sides by $(x+1)(x+2)(x+5)$ to get

$$1 = A(x+2)(x+5) + B(x+1)(x+5) + C(x+1)(x+2).$$

Substituting $x=-1$, $x=-2$, $x=-5$ into this we get the three equations

$$1 = A(1)(4)+0+0 \Rightarrow A = 1/4$$

$$1 = 0+B(-1)(3)+0 \Rightarrow B = -1/3$$

$$1 = 0+0+C(-4)(-3) \Rightarrow C = 1/12$$

#10 continued. We have shown,

$$\frac{1}{x^3+8x^2+17x+10} = \frac{1/4}{x+1} + \frac{-1/3}{x+2} + \frac{1/12}{x+5}.$$ Therefore

$$\int \frac{1}{x^3+8x^2+17x+10}\,dx = \frac{1}{4}\int \frac{1}{x+1}\,dx - \frac{1}{3}\int \frac{1}{x+2}\,dx + \frac{1}{12}\int \frac{1}{x+5}\,dx$$

$$= \frac{1}{4}\ln|x+1| - \frac{1}{3}\ln|x+2| + \frac{1}{12}\ln|x+5| + C.$$

$$\boxed{\int \frac{1}{x^3+8x^2+17x+10}\,dx = \frac{1}{4}\ln|x+1| - \frac{1}{3}\ln|x+2| + \frac{1}{12}\ln|x+5| + C}$$

(answer).

Note The answers to #9 and #10 can also be written as:

#9 $$\boxed{\int \frac{2x}{x^2-3x-10}\,dx = \ln\left(\frac{(x-5)^{10/7}}{(x+2)^{4/7}}\right) + C}$$ (answer)

#10 $$\boxed{\int \frac{1}{x^3+8x^2+17x+10}\,dx = \ln\left|\frac{(x+1)^{1/4}(x+5)^{1/12}}{(x+2)^{1/3}}\right| + C}$$ (answer)