(1) First rewrite y as

$$y = \frac{4}{\sqrt{x^3}} = \frac{4}{(x^3)^{1/2}} = \frac{4}{x^{3/2}} = 4x^{-3/2}.$$

So $$\frac{dy}{dx} = 4\left(-\frac{3}{2}\right)x^{-\frac{3}{2}-1} = \frac{-12}{2}x^{-5/2} = \frac{-6}{x^{5/2}}.$$

Therefore $$\boxed{\frac{dy}{dx} = \frac{-6}{\sqrt{x^5}}}$$

(2) Use the quotient rule:

$$\frac{dy}{dx} = \frac{(\sin x)'(x^5) - (\sin x)(x^5)'}{(x^5)^2}$$

$$= \frac{(\cos x)(x^5) - (\sin x)(5x^4)}{x^{10}}$$

$$= \frac{x^4(x\cos x - 5\sin x)}{x^{10}}$$

So $$\boxed{\frac{dy}{dx} = \frac{x\cos x - 5\sin x}{x^6}}$$

(3) Use the quotient rule:

$$\frac{dy}{dx} = \frac{(\ln x)' e^x - (\ln x)(e^x)'}{(e^x)^2}$$

$$= \frac{\left(\frac{1}{x}\right)e^x - (\ln x)(e^x)}{(e^x)^2}$$

$$= \frac{e^x\left(\frac{1}{x} - \ln x\right)}{(e^x)^2}$$

$$= \frac{\frac{1}{x} - \ln x}{e^x}$$

So $\boxed{\frac{dy}{dx} = \frac{1 - x\ln x}{xe^x}}$

(4) Use the product rule:

$$\frac{dy}{dx} = (x^2+1)'(\sin x) + (x^2+1)(\sin x)'$$

$$= (2x)(\sin x) + (x^2+1)(\cos x)$$

So $\boxed{\frac{dy}{dx} = 2x\sin x + (\cos x)(x^2+1)}$

(5) Rewrite y as $y = (\cos(x^3+1))^5$.
Let $u = \cos(x^3+1)$. So $y = u^5$.
Use the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (5u^4)(-\sin(x^3+1)(3x^2))$$

$$= -15u^4 x^2 \sin(x^3+1)$$

$$= -15x^2 (\cos(x^3+1))^4 \sin(x^3+1).$$

So $\boxed{\frac{dy}{dx} = -15x^2 \cos^4(x^3+1)\sin(x^3+1).}$

(6) Use the chain rule. Let $u = \sin x$.
So $y = e^u$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (e^u)(\cos x) = e^{\sin x} \cos x.$$

So $\boxed{\frac{dy}{dx} = e^{\sin x} \cos x}$