(#1) $f(x) = x^7(1-x^4)^{5/3}$

Use the product rule.

$$f'(x) = (x^7)'(1-x^4)^{5/3} + x^7\left((1-x^4)^{5/3}\right)'$$

$$= 7x^6(1-x^4)^{5/3} + x^7\left(\frac{5}{3}\right)(1-x^4)^{2/3}(-4x^3)$$

$$= 7x^6(1-x^4)^{5/3} - \frac{20}{3}x^{10}(1-x^4)^{2/3}$$

$$= x^6(1-x^4)^{2/3}\left(7(1-x^4) - \frac{20}{3}x^4\right)$$

$$= \frac{1}{3}(1-x^4)^{2/3}\left(21(1-x^4) - 20x^4\right)$$

$$= \frac{1}{3}(1-x^4)^{2/3}(21 - 41x^4)$$

$$\boxed{f'(x) = \frac{1}{3}(1-x^4)^{2/3}(21-41x^4)}$$

We can factor the answer even more since

$$1-x^4 = (1-x^2)(1+x^2) = (1-x)(1+x)(1+x^2)$$

So the final answer is:

$$\boxed{f'(x) = \frac{1}{3}(1-x)^{2/3}(1+x)^{2/3}(1+x^2)^{2/3}(21-41x^4)}$$

#2

$$f(x) = \frac{x + x^2}{\cos x}$$

Use the quotient rule

$$f'(x) = \frac{(x+x^2)'\cos x - (x+x^2)(\cos x)'}{(\cos x)^2}$$

$$f'(x) = \frac{(1+2x)\cos x - (x+x^2)(-\sin x)}{\cos^2 x}$$

$$\boxed{f'(x) = \frac{(1+2x)\cos x + (x+x^2)\sin x}{\cos^2 x}}$$

#3

$$f(x) = \sqrt[3]{\tan(\sin^2 x)} = (\tan(\sin^2 x))^{1/3}$$

Use the chain rule,

$$\boxed{f'(x) = \frac{1}{3}(\tan(\sin^2 x))^{-2/3}(\sec^2(\sin^2 x))(2\sin x(\cos x))}$$

#4 Must use implicit differentiation to find $\frac{dy}{dx}$.

$$x^3y^2 = 2y - y^2$$

$$(3x^2)y^2 + x^3(2y)\left(\frac{dy}{dx}\right) = 2\left(\frac{dy}{dx}\right) - 2y\left(\frac{dy}{dx}\right)$$

$$2y\left(\frac{dy}{dx}\right) + 2x^3y\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right) = -3x^2y^2$$

$$\left(\frac{dy}{dx}\right)(2y + 2x^3y - 2) = -3x^2y^2$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2y^2}{2y + 2x^3y - 2}}$$

The slope of the tangent line at the point $(1,1)$ is

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-3(1)^2(1)^2}{2(1) + 2(1)^3(1) - 2} = \frac{-3}{2}$$

The tangent line at $(1,1)$ is:

$$y - y_0 = m(x - x_0)$$

$$y - 1 = \frac{-3}{2}(x-1)$$

$$y - 1 = \frac{-3}{2}x + \frac{3}{2}$$

$$\boxed{y = \frac{-3}{2}x + \frac{5}{2}}$$

#5 First simplify,

$$\frac{\frac{1}{x^3} - \frac{1}{8}}{x-2} = \frac{\frac{8-x^3}{8x^3}}{x-2} = \frac{8-x^3}{8x^3(x-2)}$$

$$= \frac{2^3 - x^3}{8x^3(x-2)} = \frac{(2-x)(4+2x+x^2)}{8x^3(x-2)}$$

$$= \frac{(-1)(4+2x+x^2)}{8x^3} = \frac{x^2+2x+4}{-8x^3}$$

So $$\lim_{x \to 2} \frac{\frac{1}{x^3} - \frac{1}{8}}{x-8} = \lim_{x\to 2} \frac{x^2+2x+4}{-8x^3} = \frac{4+4+4}{-8(8)} = \boxed{\frac{-3}{16}}$$

#6

$$\lim_{x\to 1^-} \frac{|x-2|}{x^2-5x+4} = \lim_{x\to 1^-} \frac{|x-2|}{(x-1)(x-4)}$$

$$= \lim_{x\to 1^-} \left(\frac{|x-2|}{x-4}\right)\left(\frac{1}{x-1}\right).$$

We know $\displaystyle\lim_{x\to 1^-} \frac{|x-2|}{x-4} = \frac{|1-2|}{1-4} = \frac{1}{-3} = \frac{-1}{3}$

and $\displaystyle\lim_{x\to 1^-} \frac{1}{x-1} = \text{“}\frac{1}{\text{small negative number}}\text{”} = -\infty$

So $\displaystyle\lim_{x\to 1^-} \frac{|x-2|}{x^2-5x+4} = \left(\frac{-1}{3}\right)(-\infty) = \boxed{+\infty}$

#7a

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)+3} - \sqrt{2x+3}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{2x+2h+3} - \sqrt{2x+3}}{h}\right)\left(\frac{\sqrt{2x+2h+3} + \sqrt{2x+3}}{\sqrt{2x+2h+3} + \sqrt{2x+3}}\right)$$

7a continued

$$f'(x) = \lim_{h\to 0} \frac{\cancel{(2x+2h+3)} - \cancel{(2x+3)}}{h\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2x+2h+3}+\sqrt{2x+3}}$$

$$= \frac{2}{\sqrt{2x+2(0)+3}+\sqrt{2x+3}}$$

$$= \frac{2}{\sqrt{2x+3}+\sqrt{2x+3}}$$

$$= \frac{2}{2\sqrt{2x+3}}$$

$$= \boxed{\frac{1}{\sqrt{2x+3}}}$$

(7b.) The slope of the tangent line at the point (3,3) is

$$m = f'(3) = \frac{1}{\sqrt{2(3)+3}} = \frac{1}{\sqrt{9}} = \frac{1}{3}.$$

The equation of the tangent line at (3,3) is:

$$y - y_0 = m(x - x_0)$$

$$y - 3 = \frac{1}{3}(x-3)$$

$$y - 3 = \frac{1}{3}x - 1$$

$$\boxed{y = \frac{1}{3}x + 2}$$